

General Certificate of Education
June 2008
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 2

MPC2

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Write $\sqrt{x^3}$ in the form x^k , where k is a fraction. (1 mark)

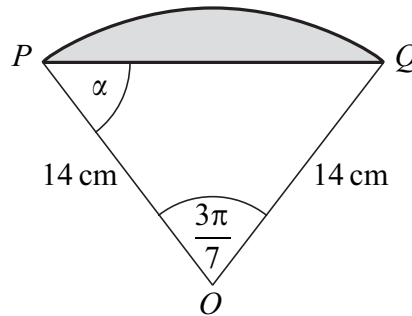
(b) A curve, defined for $x \geq 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\frac{dy}{dx}$. (3 marks)

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. (5 marks)

2 The diagram shows a shaded segment of a circle with centre O and radius 14 cm, where PQ is a chord of the circle.



In triangle OPQ , angle $POQ = \frac{3\pi}{7}$ radians and angle $OPQ = \alpha$ radians.

(a) Find the length of the arc PQ , giving your answer as a multiple of π . (2 marks)

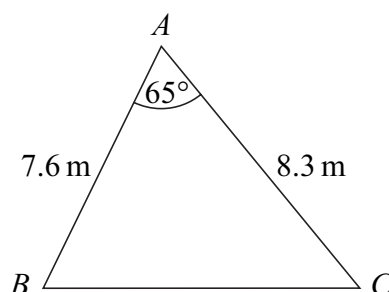
(b) Find α in terms of π . (2 marks)

(c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. (2 marks)

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \dots$$

- (a) Find the common ratio of the series. (1 mark)
- (b) Find the sum to infinity of the series. (2 marks)
- (c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. (2 marks)
- (d) Prove that the n th term of the series is 25×0.8^n . (2 marks)

4 The diagram shows a triangle ABC .

The size of angle BAC is 65° , and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

- (a) Show that the length of BC is 8.56 m, correct to three significant figures. (3 marks)
- (b) Calculate the area of triangle ABC , giving your answer in m^2 to three significant figures. (2 marks)
- (c) The perpendicular from A to BC meets BC at the point D .

Calculate the length of AD , giving your answer to the nearest 0.1 m. (3 marks)

5 (a) Write down the value of:

(i) $\log_a 1$; (1 mark)

(ii) $\log_a a$. (1 mark)

- (b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of x . (3 marks)

Turn over ►

6 The n th term of a sequence is u_n .

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \quad u_2 = 8 \quad u_3 = 4$$

- (a) Show that $q = 6$ and find the value of p . (5 marks)
- (b) Find the value of u_4 . (1 mark)
- (c) The limit of u_n as n tends to infinity is L .
- (i) Write down an equation for L . (1 mark)
- (ii) Hence find the value of L . (2 marks)

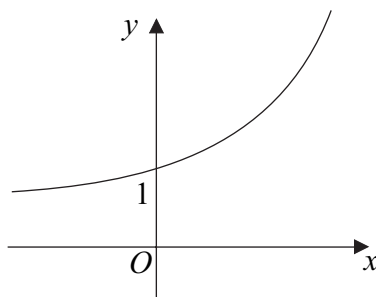
7 (a) The expression $\left(1 + \frac{4}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q . (3 marks)

- (b) (i) Hence find $\int \left(1 + \frac{4}{x^2}\right)^3 dx$. (4 marks)
- (ii) Hence find the value of $\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 dx$. (2 marks)

- 8 The diagram shows a sketch of the curve with equation $y = 6^x$.



- (a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x dx$, giving your answer to three significant figures. (4 marks)
- (ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x dx$. (2 marks)
- (b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$. (2 marks)
- (ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A . By using logarithms, find the x -coordinate of A , giving your answer to three decimal places. (4 marks)
- (c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = f(x)$. Write down an expression for $f(x)$. (2 marks)
- 9 (a) Solve the equation $\sin 2x = \sin 48^\circ$, giving the values of x in the interval $0^\circ \leq x < 360^\circ$. (4 marks)
- (b) Solve the equation $2 \sin \theta - 3 \cos \theta = 0$ in the interval $0^\circ \leq \theta < 360^\circ$, giving your answers to the nearest 0.1° . (4 marks)

END OF QUESTIONS

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