General Certificate of Education June 2007 Advanced Subsidiary Examination

MATHEMATICS Unit Pure Core 2

MPC2



Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have:

• an 8-page answer book

• the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer all questions.

1 (a) Simplify:

(i)
$$x^{\frac{3}{2}} \times x^{\frac{1}{2}}$$
; (1 mark)

(ii)
$$x^{\frac{3}{2}} \div x$$
; (1 mark)

(iii)
$$\left(\frac{3}{x^2}\right)^2$$
. (1 mark)

(b) (i) Find
$$\int 3x^{\frac{1}{2}} dx$$
. (3 marks)

(ii) Hence find the value of
$$\int_{1}^{9} 3x^{\frac{1}{2}} dx$$
. (2 marks)

2 The *n*th term of a geometric sequence is u_n , where

$$u_n = 3 \times 4^n$$

(a) Find the value of u_1 and show that $u_2 = 48$. (2 marks)

- (b) Write down the common ratio of the geometric sequence. (1 mark)
- (c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k 4$, where k is an integer. (3 marks)

(ii) Hence find the value of
$$\sum_{n=2}^{12} u_n$$
. (1 mark)

3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.



The length of the arc AB is 28 cm.

- (a) Show that $\theta = 1.4$. (2 marks)
- (b) Find the area of the sector OAB.
- (c) The point D lies on OA. The region bounded by the line BD, the line DA and the arc AB is shaded.



The length of OD is 15 cm.

- (i) Find the area of the shaded region, giving your answer to three significant figures. (3 marks)
- (ii) Use the cosine rule to calculate the length of *BD*, giving your answer to three significant figures. (3 marks)

(2 marks)

4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

- (a) Show that a + 14d = 38. (3 marks)
- (b) The sum of the second term and the seventh term is 13.Find the value of *a* and the value of *d*. (4 marks)
- 5 A curve is defined for x > 0 by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point *P* lies on the curve where x = 2.

(a) Find the *y*-coordinate of *P*. (1 mark)

(b) Expand
$$\left(1+\frac{2}{x}\right)^2$$
. (2 marks)

(c) Find
$$\frac{dy}{dx}$$
. (3 marks)

- (d) Hence show that the gradient of the curve at P is -2. (2 marks)
- (e) Find the equation of the normal to the curve at *P*, giving your answer in the form x + by + c = 0, where *b* and *c* are integers. (4 marks)

6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the *y*-coordinate of the point *A*. (2 marks)

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value
for
$$\int_0^6 3(2^x + 1) dx$$
. (4 marks)

- (c) The line y = 21 intersects the curve $y = 3(2^x + 1)$ at the point *P*.
 - (i) Show that the *x*-coordinate of *P* satisfies the equation

$$2^x = 6 \tag{1 mark}$$

(ii) Use logarithms to find the *x*-coordinate of *P*, giving your answer to three significant figures. (3 marks)

Turn over for the next question

- 7 (a) Sketch the graph of $y = \tan x$ for $0^{\circ} \le x \le 360^{\circ}$. (3 marks)
 - (b) Write down the **two** solutions of the equation $\tan x = \tan 61^\circ$ in the interval $0^\circ \le x \le 360^\circ$. (2 marks)
 - (c) (i) Given that $\sin \theta + \cos \theta = 0$, show that $\tan \theta = -1$. (1 mark)
 - (ii) Hence solve the equation $sin(x 20^\circ) + cos(x 20^\circ) = 0$ in the interval $0^\circ \le x \le 360^\circ$. (4 marks)
 - (d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x 20^\circ)$. (2 marks)
 - (e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation y = f(x). Write down an expression for f(x). (1 mark)
- 8 (a) It is given that *n* satisfies the equation

$$\log_a n = \log_a 3 + \log_a (2n - 1)$$

Find the value of *n*.

- (b) Given that $\log_a x = 3$ and $\log_a y 3\log_a 2 = 4$:
 - (i) express x in terms of a; (1 mark)

(3 marks)

(ii) express xy in terms of a. (4 marks)

END OF QUESTIONS

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8