General Certificate of Education

Mathematics 6360

MPC2 Pure Core 2

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC2

Q	Solution	Marks	Total	Comments
1(a)(i)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		Stated or explicitly used
	$= \frac{1}{2}\times 15^2 \times 1.2 = 135$ (cm^2)	A1	2	AG Must see some substitution
(ii)	{Arc =} $r\theta$	M1		PI
	…. = 18 (cm)	A1	2	
(b)	$PB = 5$ (cm)	B1		Accept even if only on a diagram or within an expression for the perimeter
	$\{AP^2 =\}\ 15^2 + 10^2 - 2\times 15\times 10\cos 1.2$	M1		RHS of cosine rule used
	$= 325 - 300\cos 1.2 = 216.2926\ldots$	m1		Correct order of evaluation
	$AP = 14.7(068..)$	A1		PI eg within an expression for perimeter
	Perimeter = 5 + 18 + 14.7…. = 37.7 (cm)	A1	5	3sf or better
	Total		**9**	
2(a)	$\sqrt{x^5} = x^{\frac{5}{2}}$	B1	1	Accept $k = 2.5$
(b)	$\int\left(7\sqrt{x^5} - 4\right)\mathrm{d}x = \frac{7}{3.5}x^{3.5} - 4x\ (+c)$	M1		Index 'k' raised by 1 in integrating x^k
		A1F		1st term correct follow through on non-integer k
		B1	3	For $-4x$ as integral of -4
(c)	$y = 2x^{3.5} - 4x + c$ (*)	B1F		y = c's answer to (b) with '+ c' ('y =' PI by next line)
	When $x = 1$, $y = 3 \Rightarrow 3 = 2 - 4 + c$	M1		Subst. (1, 3) in attempt to find constant of integration
	$y = 2x^{3.5} - 4x + 5$	A1	3	Accept $c = 5$ after correct eqn * which must include 'y =' Coefficients must be tidied
	Total		**7**	
3(a)(i)	(x =) 1	B1	1	CAO
(ii)	(x =) 3	B1	1	CAO
(b)	$\log_a n^2 = \log_a 18(n-4)$	M1		A valid law of logs applied to correct logs
		M1		A second valid law of logs applied to correct logs
	$n^2 - 18n + 72 = 0$	A1		ACF of these terms eg $n^2 - 18n = -72$
	$(n-6)(n-12) = 0$	m1		Valid method to solve quadratic, dep on both the previous Ms
	$n = 6$, $n = 12$	A1	5	Both values required **SC** NMS max (out of 5) B3 for both 6 and 12 without uniqueness considered; max B1 for either 6 or 12 only
	Total		**7**	

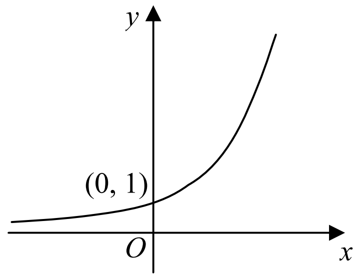
MPC2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\{S_{31} =\}\ \frac{31}{2}\left[2a+(31-1)d\right]$	M1		
	$31\left(a+15d\right)=310$	m1		Forming eqn and eliminating fraction or bracket
	$a + 15d = 310/31$; $a + 15d = 10$	A1	3	AG Completion to printed answer
(b)	$a + (21 - 1)d = 2[a + (16 - 1)d]$	M1		$a + (n - 1)d$ used for at least one term
	$\Rightarrow a = -10d$; $\Rightarrow -10d + 15d = 10$	m1		Solving $a + 15d = 10$ simultaneously with an eqn in a and d obtained from $a+20d = k[a+15d]$ with $k=2$ or with $k=1/2$
	$d = 2$	A1	3	
(c)	$u_1 = a = -20$	B1F		ft on c's value for d in $a + 15d = 10$ or in another correct (dep on m1) equation in a and d The value for a must appear within c's soln for (c)
	$\sum_{n=1}^{k} u_n = S_k = \frac{k}{2}\left[2a+(k-1)d\right]$	M1		Condone n for k in M1 and A1F lines provided n replaced by k at a later stage
	$\frac{k}{2}\left[-40+2k-2\right]=0$	A1F		'= 0' can be implied by later line; ft on c's non-zero values for a and d
	$k = 21$	A1	4	Condone presence of $k = 0$ **SC** NMS $k = 21$ and with $d = 2$ found earlier award B2. If $k = 21$ but never see $d = 2$, award 0/4
	Total		**10**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{1}{x^3} = x^{-3}$	B1		PI by its correct derivative
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3x^{-4} + 48$	M1		A power decreased by 1; could be the +48 or the ft after B0
		A1	3	
(b)	$-3x^{-4} + 48 = 0$	M1		c's answer to (a) equated to 0
	$x^{-4} = 16$	A1F		To $x^p = q$ but only ft on eqns of the form $ax^{2k} + 48 = 0$, where a and k are **negative integers**
	$x = \pm\frac{1}{2}$	A1		
	Eqns of tangents: $y = 32$ and $y = -32$	A1F	4	Only ft if answer is of the form $y = \pm k$
(c)	When $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -3 + 48 = 45$	M1		Attempt to find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 1$
	Gradient of normal at (1, 49) is $-\frac{1}{45}$	m1		Correct use of $m \times m' = -1$ with c's value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1$
	Normal at (1, 49): $y - 49 = -\frac{1}{45}(x-1)$	A1	3	CSO. Apply ISW after ACF; accept 49.02 or better in place of $49\frac{1}{45}$
	Total		**10**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
6(a)		B1 B1	 2	Shape with some indication of asymptotic behaviour in 2nd quadrant below pt of intersection with y-axis Only intersection is with y-axis at (0, 1) stated/indicated … (accept 1 on y-axis as equivalent)
(b)(i)	$h = 0.5$ $f(x) = 2^x$ $I \approx h/2\{\ldots\}$ $\{\ldots\} = f(0)+f(2)+2[f(0.5)+f(1)+f(1.5)]$	B1 M1		PI OE summing of areas of the 4 'trapezia'
	$\{\ldots\} = 1 + 4 + 2(\sqrt{2} + 2 + \sqrt{8})$ $= 5 + 2 \times 6.2426\ldots = 17.485\ldots$	A1		OE Accept 2dp (rounded or truncated) as evidence for surds
	$(I \approx)$ $4.3713\ldots = 4.37$ (to 3sf)	A1	4	CAO Must be 4.37 **SC** for those who use 5 strips, max possible is B0M1A1A0
(ii)	Increase the number of ordinates	E1	1	OE
(c)	Translation;	B1;		Accept 'translat…' as equivalent [T or Tr is NOT sufficient]
	$\begin{bmatrix} -7 \\ 3 \end{bmatrix}$	B1;B1	3	B1 for each component of the vector. Condone if the equiv 2 vectors are given. Accept **full** equivalent to vector(s) in words provided linked to 'translation/ move/shift' and **correct** directions. (No marks if **different** transformations)
(d)	$8 = 2^k + 3 \Rightarrow 2^k = 5$	M1		Correct subst. and an attempted rearrangement to $2^k = N$. PI by $k = \dfrac{\log 5}{\log 2}$
	$k = \log_2 5$	A1	2	Accept $m = 2$, $n = 5$
	Total		**12**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$(1+2x)^7$ $=1+\binom{7}{1}(2x)^1+\binom{7}{2}(2x)^2+\binom{7}{3}(2x)^3+$	M1		Any valid method. PI by a correct value for either a or b or c
	$= 1 + 14x + 84x^2 + 280x^3 + \ldots.$ $\{a = 14,\ b = 84,\ c = 280\}$	A1 × 3	4	A1 for each of a, b, c **SC** $a = 7$, $b = 21$, $c = 35$ either explicitly or within expn (M1A0)
(b)	$\left(1-\frac{1}{2}x\right)^2 = 1 - x + \frac{1}{4}x^2$	B1		Correct expansion stated explicitly or used later
	x^3 terms from expn of $\left(1-\frac{1}{2}x\right)^2(1+2x)^7$ are cx^3 and $-x\left(bx^2\right)$ and $\frac{1}{4}x^2(ax)$	M1		Any one of the three, or ft on c's non-zero values for a, b or c. Must be from products of terms using c's two expansions
	$cx^3 - x\left(bx^2\right) + \frac{1}{4}x^2(ax)$	A1F		ft c's two expansions provided all three combinations of terms are present
	Coefficient of x^3 is $c - b + 0.25a = 199.5$	A1	4	OE eg 399/2 Condone $199.5x^3$
	Total		**8**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$x + 52^\circ = (22^\circ)$, $180^\circ + 22^\circ$; $360^\circ + 22^\circ$ $(x = 180 + 22 - 52;\ x = 360 + 22 - 52)$	M1;M1		$x + 52 = 180$ + AWRT 22, 360 + AWRT 22 OE (max of M1 if extras in range) LHS could be any letter but not x unless final answer shows recovery Ms can be PI
	$x = 150^\circ$, 330°	A1	3	Both CAO with no extras in $0^\circ \le x \le 360^\circ$ Ignore anything outside $0^\circ \le x \le 360^\circ$
(b)(i)	$3\tan\theta = \frac{8}{\sin\theta} \Rightarrow 3\frac{\sin\theta}{\cos\theta} = \frac{8}{\sin\theta}$	M1		$\tan\theta = \frac{\sin\theta}{\cos\theta}$ used/seen
	$\frac{3(1-\cos^2\theta)}{\cos\theta} = 8$	M1		$\sin^2\theta = 1 - \cos^2\theta$ **used**
	$\Rightarrow 3 - 3\cos^2\theta = 8\cos\theta$ $\Rightarrow 3\cos^2\theta + 8\cos\theta - 3 = 0$	A1	3	CSO AG Completion
(ii)	$(3\cos\theta - 1)(\cos\theta + 3) = 0$	M1		Any valid method to solve the quadratic
	$\cos\theta = \frac{1}{3}$	A1	2	CSO Must only be the one value
(iii)	$\cos 2x = \frac{1}{3}$	M1		Using (ii) OE to get or use $\cos 2x = k$ where $-1 \le k \le 1$
	$(2x =)$ 70.528..	B1		Award for $\cos^{-1}(1/3)$ = value from 70 to 71 inclusive, even if θ used. PI
	$2x = 360^\circ - 70.528..$ $(= 289.47\ldots)$	m1		$2x = 360 - \cos^{-1}$(c's k) OE No extras inside the range
	$x = 35^\circ$, 145° (to the nearest degree)	A1	4	Both, condoning greater accuracy, with no extras in $0^\circ \le x \le 180^\circ$ Ignore anything outside $0^\circ \le x \le 180^\circ$ **SC for (b)(iii) only when c's answer for (b)(ii) is** $\boldsymbol{\cos\theta = -\frac{1}{3}}$**:** max mark M1B1 (val 70-71 or val 109-110 inclusive) m1A0
	Total		**12**	
	TOTAL		**75**	