# Mark Scheme (Results)
# Summer 2008

## GCE Mathematics (6663/01)

GCE

# June 2008
# 6663 Core Mathematics C1
# Mark Scheme

| Question number | Scheme | Marks |
|---|---|---|
| 1. | $2x + \frac{5}{3}x^3 + c$ | M1A1A1 |
| | | (3) |
| | | **3** |

M1 for an attempt to integrate $x^n \to x^{n+1}$. Can be given if $+c$ is only correct term.

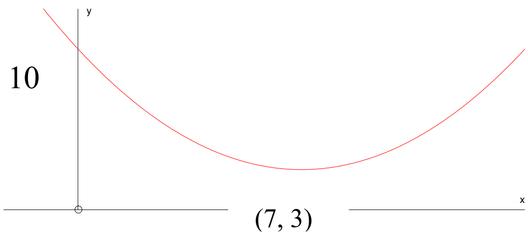
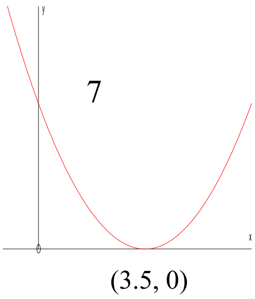
1st A1 for $\frac{5}{3}x^3$ or $2x + c$. Accept $1\frac{2}{3}$ for $\frac{5}{3}$. Do <u>not</u> accept $\frac{2x}{1}$ or $2x^1$ as final answer

2nd A1 for as printed (no extra or omitted terms). Accept $1\frac{2}{3}$ or $1.\dot{6}$ for $\frac{5}{3}$ but not 1.6 or 1.67 etc

Give marks for the first time correct answers are seen e.g. $\frac{5}{3}$ that later becomes 1.67, the 1.67 is treated as ISW

NB M1A0A1 is not possible

| Question number | Scheme | Marks |
|---|---|---|
| 2. | $x(x^2-9)$ or $(x\pm 0)(x^2-9)$ or $(x-3)(x^2+3x)$ or $(x+3)(x^2-3x)$ | B1 |
| | $x(x-3)(x+3)$ | M1A1 (3) |
| | | **3** |
| | B1 for first factor taken out correctly as indicated in line 1 above. So $x(x^2+9)$ is B0<br>M1 for attempting to factorise a relevant quadratic.<br>"Ends" correct so e.g. $(x^2-9)=(x\pm p)(x\pm q)$ where $pq$ = 9 is OK.<br>This mark can be scored for $\left(x^2-9\right)=\left(x+3\right)\left(x-3\right)$ seen anywhere.<br>A1 for a fully correct expression with all 3 factors.<br>Watch out for $-x(3-x)(x+3)$ which scores A1<br>Treat any working to solve the equation $x^3-9x$ as ISW. | |

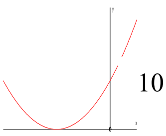
| Question number | Scheme | Marks |
|---|---|---|
| 3 | (a)   | B1B1B1 (3) |
| | (b)   | B1B1 (2) |
| | | **5** |

Allow "stopping at" (0, 10) or (0, 7) instead of "cutting"

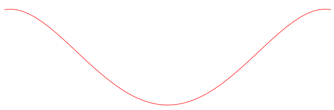
(a) 1st B1 for moving the given curve up. Must be U shaped curve, minimum in first quadrant, not touching $x$-axis but cutting positive $y$-axis. Ignore any values on axes.

2nd B1 for curve cutting $y$-axis at (0, 10) . Point 10(or even (10, 0) marked on positive $y$-axis is OK)

3rd B1 for minimum indicated at (7, 3). Must have both coordinates and in the right order.




this would score B0B1B0



If the curve flattens out to a turning point like this penalise once at first offence ie 1st B1 in (a) or in (b) but not in both.

The U shape mark can be awarded if the sides are fairly straight as long as the vertex is rounded.

(b) 1st B1 for U shaped curve, touching positive $x$-axis and crossing $y$-axis at (0, 7)[condone (7, 0) if marked on positive $y$ axis] or 7 marked on $y$-axis

2nd B1 for minimum at (3.5, 0) or 3.5 or $\frac{7}{2}$ marked on $x$-axis. Do not condone (0, 3.5) here.

Redrawing f($x$) will score B1B0 in part (b).

Points on sketch override points given in text/table.
If coordinates are given elsewhere (text or table) marks can be awarded if they are compatible with the sketch.

| Question number | Scheme | Marks |
|---|---|---|
| 4. (a) | $[\,f'(x) = \,]\ 3+3x^2$ | M1A1 (2) |
| (b) | $3+3x^2 = 15$ and start to try and simplify | M1 |
| | $x^2 = k \rightarrow x = \sqrt{k}$ (ignore $\pm$) | M1 |
| | $x = 2$ (ignore $x = -2$) | A1 (3) |
| | | **5** |
| (a) | M1 for attempting to differentiate $x^n \rightarrow x^{n-1}$. Just one term will do.<br>A poor integration attempt that gives $3x^2 + ...$ (or similar) scores M0A0<br>A1 for a fully correct expression. Must be 3 not $3x^0$. If there is a + *c* they score A0. | |
| (b) | 1st M1 for forming a correct equation and trying to rearrange their $f'(x) = 15$ e.g. collect terms.<br>e.g. $3x^2 = 15-3$ or $1+x^2 = 5$ or even 3+3$x^2 \rightarrow$ $3x^2 = \frac{15}{3}$ or $3x^{-1}+3x^2 = 15 \rightarrow 6x = 15$<br>(i.e algebra can be awful as long as they try to collect terms in their $f'(x) = 15$ equation)<br><br>2nd M1 this is dependent upon their $f'(x)$ being of the form $a+bx^2$ and attempting to solve $a+bx^2 = 15$<br>For correct processing leading to $x = \ldots$<br>Can condone arithmetic slips but processes should be correct so<br>e.g. $3+3x^2 = 15 \rightarrow 3x^2 = \frac{15}{3} \rightarrow x = \frac{\sqrt{15}}{3}$ scores M1M0A0<br>$3+3x^2 = 15 \rightarrow 3x^2 = 12 \rightarrow x^2 = 9 \rightarrow x = 3$ scores M1M0A0<br>$3+3x^2 = 15 \rightarrow 3x^2 = 12 \rightarrow 3x = \sqrt{12} \rightarrow x = \frac{\sqrt{12}}{3}$ scores M1M0A0 | |

| Question number | Scheme | Marks |
|---|---|---|
| 5. (a) | $[x_2 =]\, a-3$ | B1 (1) |
| (b) | $[x_3 =]\;\; ax_2 - 3$ or $a(a-3)-3$ | M1 |
| | $= a(a-3)-3$ } both lines needed for A1 | |
| | $= a^2 - 3a - 3$ **(*)** | A1cso (2) |
| (c) | $a^2 - 3a - 3 = 7$ | |
| | $a^2 - 3a - 10 = 0$ or $a^2 - 3a = 10$ | M1 |
| | $(a-5)(a+2) = 0$ | dM1 |
| | $\underline{a = 5 \text{ or } -2}$ | A1 (3) |
| | | **6** |
| (a) | B1 for $a \times 1 - 3$ or better. Give for $a-3$ in part (a) or if it appears in (b) they must state $x_2 = a-3$<br>This must be seen in (a) or before the $a(a-3)-3$ step. | |
| (b) | M1 for clear show that. Usually for $a(a-3)-3$ but can follow through their $x_2$ and even allow $ax_2 - 3$<br>A1 for correct processing leading to printed answer. Both lines needed and no incorrect working seen. | |
| (c) | 1st M1 for attempt to form a correct equation and start to collect terms. It must be a quadratic but need not lead to a 3TQ=0<br><br>2nd dM1 This mark is dependent upon the first M1.<br>for attempt to factorize their 3TQ=0 or to solve their 3TQ=0. The "=0"can be implied.<br>$(x \pm p)(x \pm q) = 0$, where $pq = 10$ or $(x \pm \frac{3}{2})^2 \pm \frac{9}{4} - 10 = 0$ or correct use of quadratic formula with $\pm$<br>They must have a form that leads directly to 2 values for *a*.<br>Trial and Improvement that leads to only one answer gets M0 here.<br>A1 for both correct answers. Allow $x =$…<br><br>Give 3/3 for correct answers with no working or trial and improvement that gives <u>both</u> values for *a* | |

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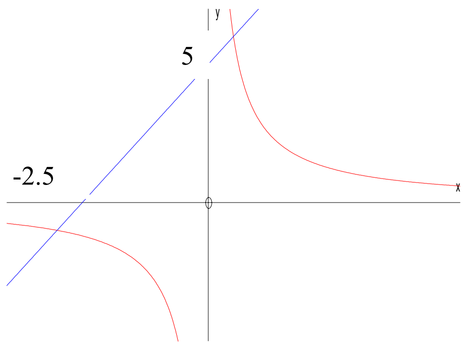
| Question Number | Scheme | Marks |
|---|---|---|
| 6. (a) |  | B1M1A1 (3) |
| (b) | $2x+5=\frac{3}{x}$ | M1 |
| | $2x^2+5x-3\,[=0]$ or $2x^2+5x=3$ | A1 |
| | $(2x-1)(x+3)\,[=0]$ | M1 |
| | $x=-3$ or $\frac{1}{2}$ | A1 |
| | $y=\frac{3}{-3}$ or $2\times(-3)+5$ or $y=\frac{3}{\frac{1}{2}}$ or $2\times\left(\frac{1}{2}\right)+5$ | M1 |
| | Points are $\underline{(-3,-1)\ \text{and}\ (\frac{1}{2},6)}$ (correct pairings) | A1ft |
| | | **9** |

(a) B1 for curve of correct shape i.e 2 branches of curve, in correct quadrants, of roughly the correct shape and no touching or intersections with axes.

Condone up to 2 inward bends but there must be some ends that are roughly asymptotic.

M1 for a straight line <u>cutting</u> the positive $y$-axis and the negative $x$-axis. Ignore any values.

A1 for (0,5) and (-2.5,0) or points correctly marked on axes. Do not give for values in tables.

Condone mixing up $(x, y)$ as $(y, x)$ if one value is zero and other value correct.

(b) 1st M1 for attempt to form a suitable equation and multiply by $x$ (at least one of $2x$ or $+5$) should be multiplied.

1st A1 for correct 3TQ - condone missing = 0

2nd M1 for an attempt to solve a relevant 3TQ leading to 2 values for $x = \ldots$

2nd A1 for both $x = -3$ and 0.5.

T&I for $x$ values <u>may</u> score 1st M1A1 otherwise no marks unless both values correct.

Answer only of $x = -3$ and $x = \frac{1}{2}$ scores 4/4, then apply the scheme for the final M1A1ft

3rd M1 for an attempt to find at least one $y$ value by substituting their $x$ in either $\frac{3}{x}$ or $2x + 5$

3rd A1ft follow through both their $x$ values, in either equation but the same for each, correct pairings required but can be $x = -3$, $y = -1$ etc

| Question number | Scheme | Marks | |
|---|---|---|---|
| **7.** (a) | 5, 7, 9, 11 or 5+2+2+2=11 or 5+6=11 use $a$ = 5, $d$ = 2, $n$ = 4 and $t_4 = 5 + 3 \times 2 = 11$ | B1 | (1) |
| (b) | $t_n = a + (n-1)d$ with one of $a$ = 5 or $d$ = 2 correct (can have a letter for the other) | M1 | |
| | $= 5 + 2(n - 1)$ or $2n + 3$ or $1 + 2(n + 1)$ | A1 | (2) |
| (c) | $S_n = \frac{n}{2}[2 \times 5 + 2(n-1)]$ or use of $\frac{n}{2}(5 + \text{"their } 2n+3\text{"})$ (may also be scored in (b)) | M1A1 | |
| | $= \{n(5 + n - 1)\} = n(n + 4)$ **(*)** | A1cso | (3) |
| (d) | $43 = 2n + 3$ | M1 | |
| | $[n] = 20$ | A1 | (2) |
| (e) | $S_{20} = 20 \times 24$, $= \underline{480}$ (km) | M1A1 | (2) |
| | | **10** | |

| | | |
|---|---|---|
| (a) | B1 | Any other sum must have a convincing argument |
| (b) | M1 | for an attempt to use $a + (n - 1)d$ with one of $a$ or $d$ correct (the other can be a letter)<br>Allow any answer of the form $2n + p$ ($p \neq 5$) to score M1. |
| | A1 | for a correct expression (needn't be simplified) [ **Beware** $5 + (2n-1)$ scores A0]<br>Expression must be in $n$ not $x$.<br>Correct answers with no working scores 2/2. |
| (c) | M1 | for an attempt to use $S_n$ formula with $a$ = 5 or $d$ = 2 or $a$ = 5 and their "$2n$ + 3" |
| | 1st A1 | for a fully correct expression |
| | 2nd A1 | for correctly simplifying to given answer. No incorrect working seen. Must see $S_n$ used. |
| | | Do not give credit for part (b) if the equivalent work is given in part (d) |
| (d) | M1 | for forming a suitable equation in $n$ (ft their (b)) and attempting to solve leading to $n$ =… |
| | A1 | for 20<br>Correct answer only scores 2/2 . Allow 20 following a restart but check working.<br>eg 43 = $2n$ + 5 that leads to 40 = $2n$ and $n$ =20 should score M1A0. |
| (e) | M1 | for using their answer for $n$ in $n(n + 4)$ or $S_n$ formula, their $n$ must be a value. |
| | A1 | for 480 (ignore units but accept 480 000 m etc)[ no matter where their 20 comes from] |
| | | NB "attempting to solve" eg part (d) means we will allow sign slips and slips in arithmetic but not in processes. So dividing when they should subtract etc would lead to M0.<br>Listing in parts (d) and (e) can score 2 (if correct) or 0 otherwise in each part.<br>Poor labelling may occur (especially in (b) and (c) ) . If you see work to get $n(n + 4)$ mark as (c) |

| Question number | Scheme | Marks |
|---|---|---|
| 8. (a) | [No real roots implies $b^2-4ac<0$ .] $b^2-4ac=q^2-4\times 2q\times(-1)$ | M1 |
| | So $q^2-4\times 2q\times(-1)<0$ i.e. $q^2+8q<0$ (*) | A1cso (2) |
| (b) | $q(q+8)=0$ or $(q\pm 4)^2\pm 16=0$ | M1 |
| | $(q)=0$ or $-8$ (2 cvs) | A1 |
| | $-8<q<0$ <u>or</u> $q\in$ (-8, 0) <u>or</u> $q<0$ and $q>-8$ | A1ft (3) |
| | | **5** |

| | |
|---|---|
| (a) | M1 for attempting $b^2-4ac$ with one of $b$ or $a$ correct. < 0 not needed for M1<br>This may be inside a square root.<br>A1cso for simplifying to printed result with no incorrect working or statements seen.<br>Need an intermediate step<br>e.g. $q^2--8q<0$ or $q^2-4\times 2q\times -1<0$ or $q^2-4(2q)(-1)<0$ or $q^2-8q(-1)<0$ or $q^2-8q\times -1<0$<br>i.e. must have × or brackets on the $4ac$ term<br>< 0 must be seen at least one line before the final answer. |
| (b) | M1 for factorizing or completing the square or attempting to solve $q^2\pm 8q=0$. A method that would lead to 2 values for $q$. The "= 0" may be implied by values appearing later.<br>1st A1 for $q=0$ and $q=-8$<br>2nd A1 for $-8<q<0$. Can follow through their cvs but must choose "inside" region.<br>$q<0, q>-8$ is A0, $q<0$ or $q>-8$ is A0, (-8, 0) on its own is A0<br>BUT "$q<0$ and $q>-8$" is A1<br><br>Do not accept a number line for final mark |

| Question number | Scheme | Marks |
|---|---|---|
| 9. (a) | $\left[\frac{dy}{dx}=\right]3kx^2-2x+1$ | M1A1 (2) |
| (b) | Gradient of line is $\frac{7}{2}$ | B1 |
| | When $x=-\frac{1}{2}$: $3k\times\left(\frac{1}{4}\right)-2\times\left(-\frac{1}{2}\right)+1,=\frac{7}{2}$ | M1, M1 |
| | $\frac{3k}{4}=\frac{3}{2}\Rightarrow k=2$ | A1 (4) |
| (c) | $x=-\frac{1}{2}\Rightarrow y=k\times\left(-\frac{1}{8}\right)-\left(\frac{1}{4}\right)-\frac{1}{2}-5,=-6$ | M1, A1 (2) |
| | | **8** |

(a) M1 for attempting to differentiate $x^n\to x^{n-1}$ (or -5 going to 0 will do)

A1 all correct. A "+ $c$" scores A0

(b) B1 for $m=\frac{7}{2}$. Rearranging the line into $y=\frac{7}{2}x+c$ does not score this mark until you are sure they are using $\frac{7}{2}$ as the gradient of the line or state $m=\frac{7}{2}$

1st M1 for substituting $x=-\frac{1}{2}$ into their $\frac{dy}{dx}$, some correct substitution seen

2nd M1 for forming a suitable equation in $k$ and attempting to solve leading to $k=\ldots$
Equation must use their $\frac{dy}{dx}$ and <u>their gradient of line</u>. Assuming the gradient is 0 or 7 scores M0 unless they have clearly stated that this is the gradient of the line.

A1 for $k = 2$

(c) M1 for attempting to substitute their $k$ (however it was found or can still be a letter) and $x=-\frac{1}{2}$ into $y$ (some correct substitution)

A1 for - 6

| Question number | Scheme | Marks |
|---|---|---|
| 10. (a) | $QR=\sqrt{(7-1)^2+(0-3)^2}$ | M1 |
| | $=\sqrt{36+9}$ or $\sqrt{45}$ (condone $\pm$) | A1 |
| | $=3\sqrt{5}$ or $a=3$ ($\pm 3\sqrt{5}$ etc is A0) | A1 (3) |
| (b) | Gradient of $QR$ (or $l_1$) $=\dfrac{3-0}{1-7}$ or $\dfrac{3}{-6}, =-\dfrac{1}{2}$ | M1, A1 |
| | Gradient of $l_2$ is $-\dfrac{1}{-\frac{1}{2}}$ or 2 | M1 |
| | Equation for $l_2$ is: $y-3=2(x-1)$ or $\frac{y-3}{x-1}=2$ [or $y=2x+1$] | M1 A1ft (5) |
| (c) | $P$ is (0, 1) (allow "$x=0, y=1$" but it must be clearly identifiable as $P$) | B1 (1) |
| (d) | $PQ=\sqrt{(1-x_P)^2+(3-y_P)^2}$ | M1 |
| | $PQ=\sqrt{1^2+2^2}=\sqrt{5}$ | A1 |
| | Area of triangle is $\frac{1}{2}QR\times PQ=\frac{1}{2}3\sqrt{5}\times\sqrt{5},=\dfrac{15}{2}$ or 7.5 | dM1, A1 (4) |
| | | **13** |

**Determinant Method**
e.g.(0+0+7) - (1+21+0)

= - 15 (o.e.)
Area = $\frac{1}{2}\left|-15\right|$, = 7.5

Rules for quoting formula: For an M mark, if a correct formula is quoted and <u>some</u> correct substitutions seen then M1 can be awarded, if no values are correct then M0. If no correct formula is seen then M1 can only be scored for a fully correct expression.

(a) M1 for attempting $QR$ or $QR^2$. May be implied by $6^2+3^2$
1st A1 for as printed or better. Must have square root. Condone $\pm$

(b) 1st M1 for attempting gradient of $QR$
1st A1 for - 0.5 or $-\frac{1}{2}$, can be implied by gradient of $l_2=2$
2nd M1 for an attempt to use the perpendicular rule on their gradient of $QR$.
3rd M1 for attempting equation of a line using $Q$ with their changed gradient.
2nd A1ft requires all 3 Ms but can ft their gradient of $QR$.

$y=2x+1$ with no working. Send to review.

(d) 1st M1 for attempting $PQ$ or $PQ^2$ follow through their coordinates of $P$
1st A1 for $PQ$ as one of the given forms.
2nd dM1 for correct attempt at area of the triangle. Follow through their value of $a$ and their $PQ$. This M mark is dependent upon the first M mark
2nd A1 for 7.5 or some exact equivalent. Depends on both Ms. Some working must be seen.

<u>ALT</u> Use $QS$ where $S$ is (1, 0)
1st M1 for attempting area of $OPQS$ and $QSR$ and $OPR$. Need all 3.
1st A1 for $OPQS=\frac{1}{2}(1+3)\times 1=2$, $QSR=9$, $OPR=\frac{7}{2}$
2nd dM1 for $OPQS+QSR-OPR=\ldots$Follow through their values.
2nd A1 for 7.5

**Determinant Method**
M1 for attempt -at least one value in each bracket correct .
A1 if correct ($\pm$ 15)
M1 for correct area formula
A1 for 7.5

<u>MR</u> Misreading $x$-axis for $y$-axis for $P$. Do NOT use MR rule as this oversimplifies the question. They can only get M marks in (d) if they use $PQ$ and $QR$.

| Question number | Scheme | Marks | |
|---|---|---|---|
| 11. (a) | $\left(x^2+3\right)^2 = x^4+3x^2+3x^2+3^2$ | M1 | |
| | $\dfrac{\left(x^2+3\right)^2}{x^2} = \dfrac{x^4+6x^2+9}{x^2} = x^2+6+9x^{-2}$ **(*)** | A1cso | (2) |
| (b) | $y = \dfrac{x^3}{3} + 6x + \dfrac{9}{-1}x^{-1}\left(+c\right)$ | M1A1A1 | |
| | $20 = \dfrac{27}{3} + 6\times 3 - \dfrac{9}{3} + c$ | M1 | |
| | $c = -4$ | A1 | |
| | $[y =]\dfrac{x^3}{3} + 6x - 9x^{-1} - 4$ | A1ft | (6) |
| | | | **8** |

(a) M1 for attempting to expand $\left(x^2+3\right)^2$ and having at least 3(out of the 4) correct terms.

A1 at least this should be seen and no incorrect working seen.

If they never write $\dfrac{9}{x^2}$ as $9x^{-2}$ they score A0.

(b) 1st M1 for some correct integration, one correct $x$ term as printed or better

Trying $\dfrac{\int u}{\int v}$ loses the first M mark but could pick up the second.

1st A1 for two correct $x$ terms, un-simplified, as printed or better

2nd A1 for a fully correct expression. Terms need not be simplified and $+c$ is not required.

No $+ c$ loses the next 3 marks

2nd M1 for using $x = 3$ and $y = 20$ in their expression for f($x$) $\left[\neq \dfrac{dy}{dx}\right]$ to form a linear equation for $c$

3rd A1 for $c = -4$

4th A1ft for an expression for $y$ with simplified $x$ terms: $\dfrac{9}{x}$ for $9x^{-1}$ is OK .

Condone missing "$y =$ "

Follow through their numerical value of $c$ only.