# AQA

# Mathematics                    MPC1

**Unit Pure Core 1**

**Monday 11 January 2010    9.00 am to 10.30 am**

**For this paper you must have:**
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

**MPC1**

Answer **all** questions.

**1** The polynomial $p(x)$ is given by $p(x) = x^3 - 13x - 12$.

(a) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(3 marks)*

**2** The triangle $ABC$ has vertices $A(1, 3)$, $B(3, 7)$ and $C(-1, 9)$.

(a) (i) Find the gradient of $AB$. *(2 marks)*

(ii) Hence show that angle $ABC$ is a right angle. *(2 marks)*

(b) (i) Find the coordinates of $M$, the mid-point of $AC$. *(2 marks)*

(ii) Show that the lengths of $AB$ and $BC$ are equal. *(3 marks)*

(iii) Hence find an equation of the line of symmetry of the triangle $ABC$. *(3 marks)*

**3** The depth of water, $y$ metres, in a tank after time $t$ hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{dy}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that $y$ has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. *(4 marks)*

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. *(1 mark)*

**4** (a) Show that $\dfrac{\sqrt{50} + \sqrt{18}}{\sqrt{8}}$ is an integer and find its value. *(3 marks)*

(b) Express $\dfrac{2\sqrt{7} - 1}{2\sqrt{7} + 5}$ in the form $m + n\sqrt{7}$, where $m$ and $n$ are integers. *(4 marks)*

**5** (a) Express $(x - 5)(x - 3) + 2$ in the form $(x - p)^2 + q$, where $p$ and $q$ are integers. *(3 marks)*

(b) (i) Sketch the graph of $y = (x - 5)(x - 3) + 2$, stating the coordinates of the minimum point and the point where the graph crosses the $y$-axis. *(3 marks)*

(ii) Write down an equation of the tangent to the graph of $y = (x - 5)(x - 3) + 2$ at its vertex. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = (x - 5)(x - 3) + 2$. *(3 marks)*

**6** The curve with equation $y = 12x^2 - 19x - 2x^3$ is sketched below.
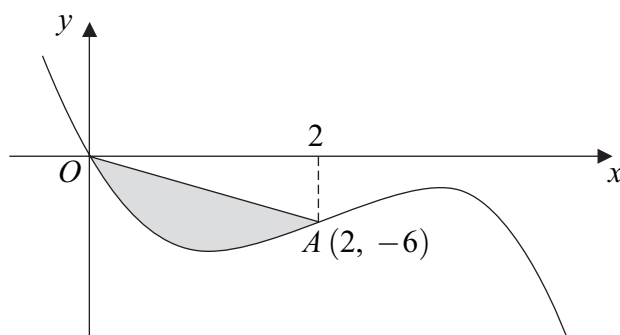


The curve crosses the $x$-axis at the origin $O$, and the point $A\,(2,\,-6)$ lies on the curve.

(a) (i) Find the gradient of the curve with equation $y = 12x^2 - 19x - 2x^3$ at the point $A$. *(4 marks)*

(ii) Hence find the equation of the normal to the curve at the point $A$, giving your answer in the form $x + py + q = 0$, where $p$ and $q$ are integers. *(3 marks)*

(b) (i) Find the value of $\displaystyle\int_0^2 (12x^2 - 19x - 2x^3)\ \mathrm{d}x$. *(5 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the line $OA$. *(3 marks)*

**Turn over for the next question**

**Turn over ▶**

**7** A circle with centre $C$ has equation $x^2 + y^2 - 4x + 12y + 15 = 0$.

   (a) Find:

      (i)   the coordinates of $C$;                                                                      *(2 marks)*

      (ii)   the radius of the circle.                                                         *(2 marks)*

   (b) Explain why the circle lies entirely below the $x$-axis.           *(2 marks)*

   (c) The point $P$ with coordinates $(5, k)$ lies outside the circle.

      (i)   Show that $PC^2 = k^2 + 12k + 45$.                       *(2 marks)*

      (ii)   Hence show that $k^2 + 12k + 20 > 0$.                     *(1 mark)*

      (iii)   Find the possible values of $k$.                         *(4 marks)*

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**END OF QUESTIONS**

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