Mechanics 1 – Revision notes

1. Kinematics in one and two dimensions

EQUATIONS FOR CONSTANT ACCELERATION ARE NOT GIVEN – Learn Them!

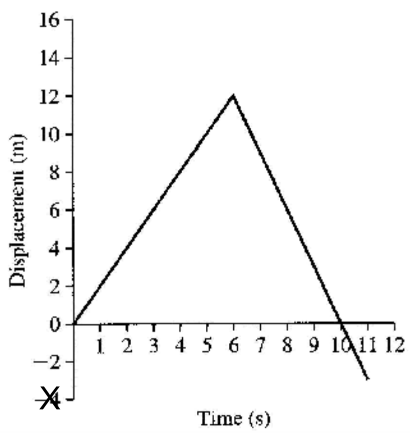
$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t \qquad v^2 = u^2 + 2as$$

s : displacement (m)
u : initial velocity (ms^{-1})
v : final velocity (ms^{-1})
a : acceleration (ms^2)
t : time (s)

- Always list the variables you have - write down the equation you intend to use.
- Sketch graphs – essential for multi-stage journeys
- Retardation / deceleration – don't forget the negative sign

Distance/ Displacement – time graph



GRADIENT = VELOCITY

Straight line – constant velocity – zero acceleration

Travels 12m from a point X, turns round and travels 15 m in the opposite direction finishing 3m behind X.

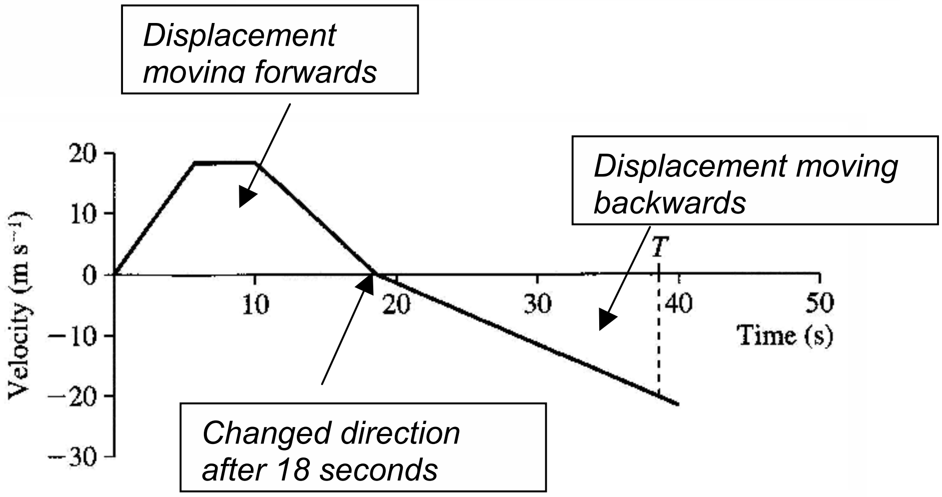
MOST USEFUL GRAPH TO SKETCH

Velocity – Time graph

GRADIENT = ACCELERATION

Straight line – constant acceleration

DISPLACEMENT is represented by the **area** under the graph



FREE FALL UNDER GRAVITY

<u>Assumptions</u>

- the body is a point mass
- air resistance can be ignored
- the motion of a body is in a vertical line
- the acceleration due to gravity is constant

Acceleration due to gravity

$9.8\ ms^{-2}$

Unless given in the question

<u>EXAMPLE</u> : A ball is thrown vertically upwards from ground level with a velocity of $28\ ms^{-1}$

a) What was its maximum height above the ground ?

$u = 28\ ms^{-1}$
$a = -9.8\ ms^{-2}$
$v = 0$ (top of balls flight)
$s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 28^2 + 2 \times (-9{\cdot}8)s$$

$$s = 40\ m$$

b) How long did it take to return to the ground ?

$u = 28\ ms^{-1}$
$a = -9.8\ ms^{-2}$
$s = 0$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 28t + \frac{1}{2}(-9{\cdot}8) \times t^2$$

$$0 = t(28 - 4{\cdot}9t)$$

$$t = 0 \text{ or } t = 5{\cdot}71$$

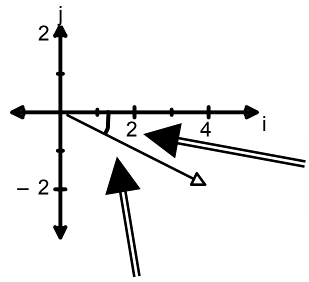
$t = 0$: time at which ball thrown

Clearly identify $t = 5.71s$ as the final answer

VECTORS

- Vectors have both magnitude and direction

$$A = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \quad \text{or} \quad A = 4i - 2j$$



Magnitude : |A| = $\sqrt{2^2 + 4^2}$

$\sqrt{20}$

- **SPEED** = magnitude of the velocity vector

Direction : $\theta = \tan^{-1}\frac{2}{4}$

$= 26{\cdot}6^\circ$

26.6º to the horizontal (i) in a negative direction

- **DIRECTION OF TRAVEL** = direction of the velocity vector

If working in **bearings** – don't forget the 3 digits e.g. 025º

- **Unit Vector :** a vector with magnitude = 1

Vector equations – for constant acceleration the 5 equations involving, displacement, velocity etc can be used

- If asked to write an equation in terms of t for displacement/ velocity etc – simplify your equation as far as possible by **collecting the i terms and j terms**

e.g $\mathbf{u} = 2i + 5j \quad \mathbf{a} = 4i - 8j$

Displacement $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{R} = (2i + 5j)t + \frac{1}{2}(4i - 8j)t^2$

$= (2t + 2t^2)i + (5t - 4t^2)j$

Example

Two particles A and B are moving in a plane with the following properties

A is at point (0,3), has velocity (2i + j) ms^{-1} and acceleration (i – 2j) ms^{-2}

B is at point (2,1), has velocity (3i - j) ms^{-1} and acceleration (2i) ms^{-2}

Find the vector $\overrightarrow{AB}$ six seconds later, and the distance between the particles at that time

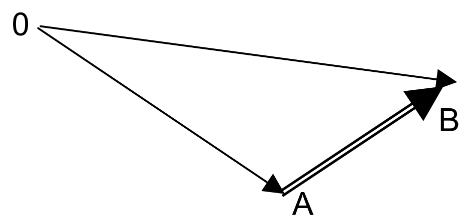
Displacement : in vector form **r** is used instead of s

Using

$$r = ut + \frac{1}{2}at^2$$

For A : $\mathbf{r} = (2i + j) \times 6 + \frac{1}{2}(i - 2j) \times 36$

$= 30i - 30j$



As A started at (0,3) six seconds later $\overrightarrow{OA} = 30i - 27j$

......$\overrightarrow{OB} = 56i - 5j$

This gives $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 26i + 22j$

Distance AB = magnitude of $\overrightarrow{AB}$

$= \sqrt{26^2 + 22^2}$

$= 34.1$ m

- Forces can be represented as vectors

- If forces are in **equilibrium** then the **resultant** (sum of vectors) = 0

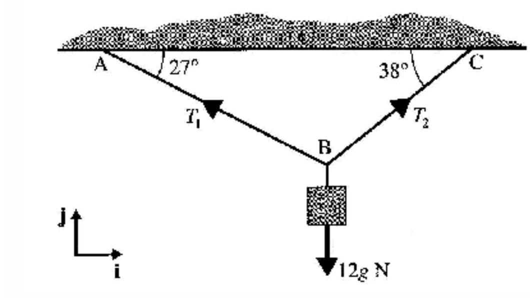
  All **i** components sum to zero and all **j** components sum to zero.

  If drawn the forces will form a **closed polygon**

<u>3 forces in equilibrium</u>

The system is in equilibrium.

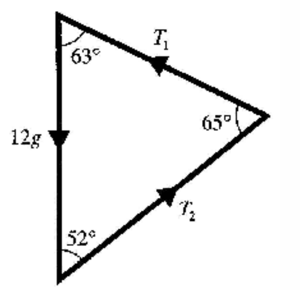
Find T_1 and T_2



Method 1 – **Triangle of forces**

Sketching the 3 forces gives a triangle.

We can now use the sine rule to find T_1 and T_2.

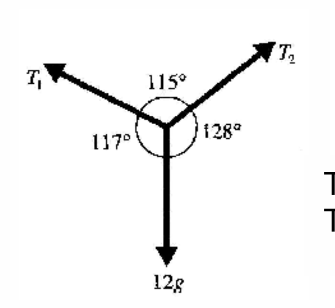


$$\frac{T_1}{\sin 52} = \frac{T_2}{\sin 63} = \frac{12g}{\sin 65}$$

$T_1 = 102$ N

$T_2 = 116$ N

Method 2 – **Lami's theorem**



$$\frac{T_1}{\sin 128} = \frac{T_2}{\sin 117} = \frac{12g}{\sin 115}$$

T_1 = 102 N
T_2 = 116 N

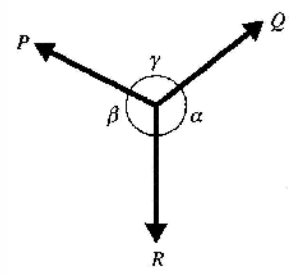
For questions involving river crossings and current you may need to use the cosine rule as well as the sine rule to calculate missing lengths and angles

Cosine Rule

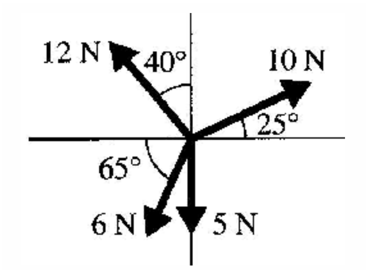
$$a^2 = b^2 + c^2 - 2bc \text{ Cos } A$$

LAMI'S THEOREM
For any set of **three** forces P,Q and R in **equilibrium**

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$



More than 3 forces in Equilibrium – Resolve the forces
Example
Find the resultant of the following system and state the force needed to maintain equilibrium.



Horizontally
Resultant (i) = -12sin 40º + 10cos 25º – 6 cos 65º
= - 1.186
Vertically
Resultant (j) = 12cos 40º + 10sin 25º – 6sin 65º – 5
= 2.981
Resultant = -1.186i + 2.981j

Force needed to maintain equilibrium = **1.186i – 2.981j**
Force of 3.21 N with direction –68.3º to the positive x-direction

$\sqrt{1{\cdot}186^2 + 2{\cdot}981^2}$ $\quad$ $\text{Tan}^{-1}\left(\frac{-2{\cdot}891}{1{\cdot}186}\right)$

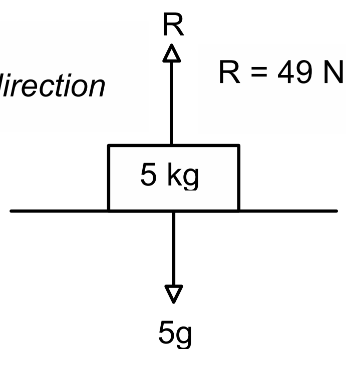
TYPES OF FORCE

- ALWAYS DRAW A DIAGRAM SHOWING ALL FORCES (with magnitude if known)

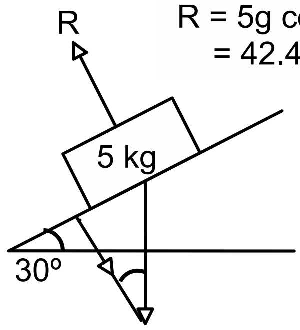
Weight : mass x 9.8 (gravity)
Reaction (normal reaction) : at right angles to the plane of contact

SYSTEMS in **Equilibrium** – resolving in the vertically (or in the j direction)

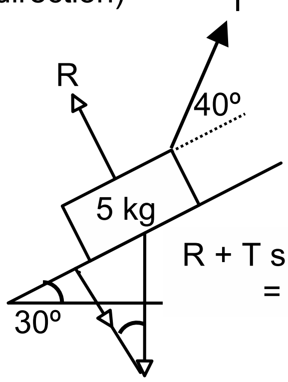
Vertical or j-direction



R = 49 N



R = 5g cos 30º
= 42.4 N



R + T sin 40º
= 5g cos 30º

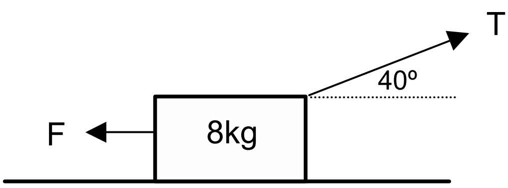
Tension / Thrust – pulling or pushing force on the body

Friction

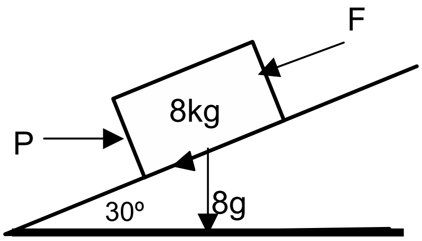
- **Always** – acts in a direction opposite to that in which the object is moving or tending to move
- **Smooth contact** – friction is small enough to be ignored
- **Maximum Friction (limiting friction) -** object is **moving** or just on the point of moving : $F = \mu R$ where μ is the coefficient of friction
- $F < \mu R$ body not moving

IN EQUILIBRIUM – resolving horizontally or in the i direction

F = T



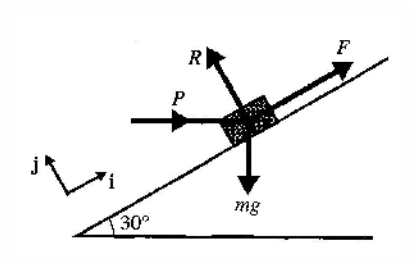
F = T Cos 40º



P Cos 30º = F + 8gSin 30º

- For questions looking for the **minimum and maximum force** needed to for a block on a slope to move look at :

A) **P is too small** – the block is about to slide down the slope (limiting friction)



Resolving in the i direction
F + Pcos 30 – mg sin 30 = 0

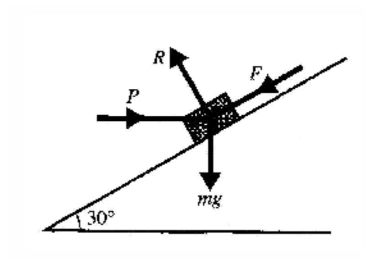
Resolving in the j direction
R – Psin 30 – mg cos 30 = 0

Change in direction of friction

B) **P is too large** – the block is on the verge of sliding up the slope

Resolving in the i direction
Pcos 30 – F – mg sin 30 = 0

Resolving in the j direction
R – Psin 30 – mg cos 30 = 0



NEWTON'S LAWS OF MOTION

1st Law Every object remains **at rest** or moves with **constant velocity** unless an external force is applied

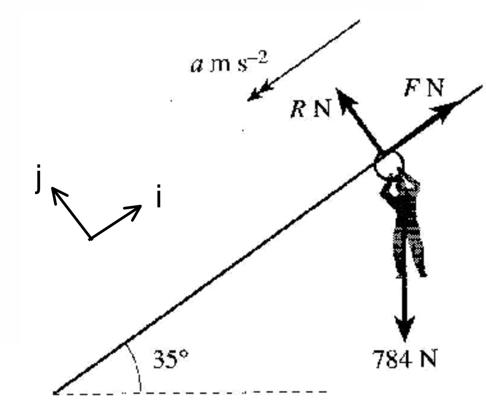
- Constant velocity - system is in **equalibrium**
  - net force (resultant force) = 0
  - in vector form – equate the i and j components to zero

2nd Law $F = ma$ **Net Force** = mass x acceleration

- Always work out and state **Net force** clearly before equating to **ma**
- Check - if acceleration is positive – net force should also be positive

Example : A taut cable 25m long is fixed at 35º to the horizontal. A light rope ring is placed around the cable at the upper end. A soldier of mass 8 kg grabs the rope ring and slides down the cable.

If the coefficient of friction between the ring and the cable is 0.4, how fast is the soldier moving when he reaches the bottom



i- direction : $784 \cos 35 = R$
j – direction : $784 \sin 35 - F = ma$

Motion – friction is limiting so F = 0.4R

$784 \sin 35 - 0.4 \times 784 \cos 35 = 80a$

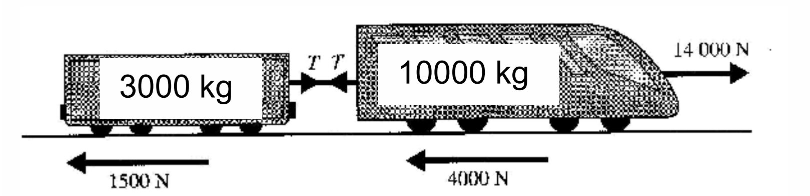
$a = 2.41 \text{ ms}^{-2}$
$u = 0$
$s = 25$

$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2 \times 2{\cdot}41 \times 25$
$v = 11{\cdot}0 \text{ ms}^{-1}$

3rd Law For every action there is an equal and opposite reaction

<u>**Connected Particles**</u>

- ***Trains and trailers***



<u>Finding the acceleration (F=ma)</u>
Net Force = 14000 – 4000 – 1500
= 8500 N

8500 = (3000+10000)a
a = 0.654 ms^{-2}

<u>Finding the Tension in the coupling</u>
- To keep it simple - use the body which has no direct force applied e.g. the trailer

Net Force = T – 1500

T - 1500 = 3000 x 0.654
T = 3451.5 N

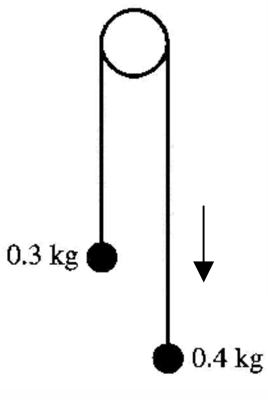
- **Stings and Pulleys**
Always draw a diagram – if known show direction of acceleration

<u>Finding the acceleration</u>

$0.4g - T = 0.4a$
$T - 0.3g = 0.3a$ (+)

$0.1g = 0.7a$

$a = 1.4 \text{ ms}^{-2}$

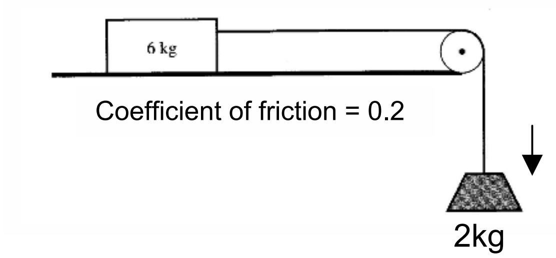


<u>Finding the tension</u>
a=1.4
substitute into either equation
(or both just to check)

T= 0.3g+0.3x1.4
T = 3.36 N

0.4g – 3.36 = 0.4 x 1.4

<u>Force on the pulley</u> = T + T



<u>Finding the acceleration</u>

$2g - T = 2a$
$T - F = 6a$

Friction = 0.2R
R = 6g so F = 11.76 N

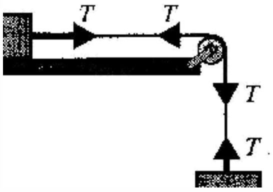
$2g - T = 2a$
$T - 11.76 = 6a$

$a = 0.98\ ms^{-2}$

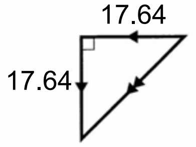
<u>Finding the Tension</u>

$T = 11.76 + 6a$
$= 17.64$ N

<u>Force on the pulley</u>



Resultant force



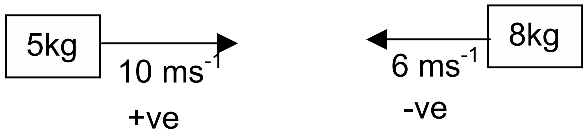
$R = \sqrt{17{\cdot}64^2 + 17{\cdot}64^2}$
$= 24{\cdot}9$ N

Acting at 45° to the horizontal

Momentum

- A quantity of motion measured in **Newton Seconds**
- **Momentum = mass x velocity**
- The total momentum of a system remains the same unless an external force is applied - **Conservation of Momentum**

Draw diagrams to show before/after masses and velocities



Momentum = 5 x 10 – 8 x 6

Example : Particle P of mass 6 kg has velocity (4i + 2j). After a collision with another particle, P has velocity (2i – 3j). Find the momentum lost by P during the collisison

Momentum of P before = 6(4i + 2j)
= 24i + 12j

Momentum of P after = 6(2i – 3j)
= 12i – 18j

Momentum lost = (24i + 12j) – (12i – 18j)
= 12i + 30j

Projectiles

- **You cannot just quote formulae – you must show how they are derived**

Max range occurs when angle = 45°

- **Initial Velocity:** $u = U\cos\theta i + U\sin\theta j$
- **Acceleration :** a = - 9.8j
- **Velocity (after t s) :** $v = (U\cos\theta i + U\sin\theta j) - 9{\cdot}8tj$

$= U\cos\theta i + (U\sin\theta - 9{\cdot}8t)j$

Particle moving in a **vertical** direction when $U\cos\theta = 0$

Particle moving in a **horizontal** direction when
$U\sin\theta - 9{\cdot}8t = 0$

$\Downarrow$

Time when particle reaches **max height**

Displacement (r) : r = ut+ ½at²

$$r = Ut\cos\theta\, i + (Ut\sin\theta - ½9{\cdot}8t^2)j$$

Horizontal dispalcement after t seconds

Height (vertical dispalcement) after t seconds

To find the range

Height component = 0
Solve

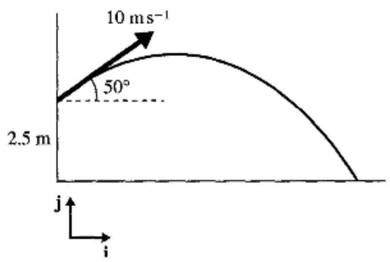
$$Ut\sin\theta - ½9{\cdot}8t^2 = 0$$

To find t

Substitute into

$$\text{Range} = Ut\cos\theta$$

Example
A shot putter releases a shot at a height of 2.5 m and with a velocity of $10ms^{-1}$ at 50° to the horizontal. Find the distance travelled by the shot.



u = 10cos50°i +10sin 50°j

Displacement from the point of projection
r = (10cos50°i +10sin 50°j)t – 4.9t²j

Displacement form the origin
r = 10tcos50°i +(2.5 + 10sin 50°t – 4.9t²)j

Height above ground (j component)
So shot hits ground when

2.5 + 10sin 50°t – 4.9t² = 0

4.9t² – 7.66t –2.5 = 0

t = -2.77 or **t = 1.84**

Horizontal distance from origin when t = 1.84 (i component)

Distance = 10 x 1.84 x cos 50°
= 11.8 m

Modelling Assumptions

Common Terms and Meanings

Term	Applies to	What is disregarded
Inextensible	Strings, rods	Stretching
Thin	Strings, rods	Diameter, thickness
Light	Strings, springs, rods	Mass
Particle	Object of negligible size	Rotational motion, size
Rigid	Rods	Bending
Small	Object of negligible size	Rotational motion
Smooth	Surfaces, pulleys	Friction

Assumptions made

- motion takes place in a straight line -
- acceleration is constant
- air resistance can be ignored
- objects are modelled as masses concentrated at a single point (no rotation)
- g is assumed to be 9.8m s^{-2} everywhere at or near the Earths surface