# Algebra : Inequalities

Symbols

$$x > y \qquad x \text{ greater than } y$$
$$x < y \qquad x \text{ less than } y$$
$$x \geq y \qquad x \text{ greater than or equal to } y$$
$$x \leq y \qquad x \text{ less than or equal to } y$$

The rules of inequalities  (sometimes called 'inequations')

These are the same as for equations i.e that whatever you do to one side of the equation(add/subtract, multiply/divide by quantities) you must do to the other.

However, their are **two** exceptions to these rules.

When you **multiply** each side **by a negative quantity**

'<' becomes '>', or '>' becomes '<' .

That is, the **inequality sign is reversed**.

Similarly, when you **divide** each side by a **negative quantity**

< becomes >, or > becomes< .

That is, the **inequality sign is reversed**.

Examples

$$-\frac{x}{2} < 6$$

multiplying each side by -2

$$\frac{-2}{-2}x > -12 \text{ (note } < \text{ to } >)$$

$$x > -12$$

$$-5x > 4$$

dividing each side by -5

$$\frac{-5}{-5}x < \frac{4}{-5} \quad \text{(note } > \text{ to } <)$$

$$x < \frac{4}{-5}$$

$$x < -\frac{4}{5}$$

Inequalities with one variable

Example #1 - Find all the integral values of x where,

$$6 \geq x > -5$$

The values of x lie equal to and less than 6 but greater than -5, but not equal to it.

The integral(whole numbers + or - or zero) values of x are therefore:

6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4

Example #2 - What is the range of values of x where,

$$x^2 \geq 144$$

Since the square root of 144 is +12 or -12(remember two negatives multiplied make a positive), x can have values between 12 and -12.

In other words the value of x is less than or equal to 12 and more than or equal to -12. This is written:

$$12 \geq x \geq -12$$

Inequalities with two variables - Solution is by arranging the equation into the form
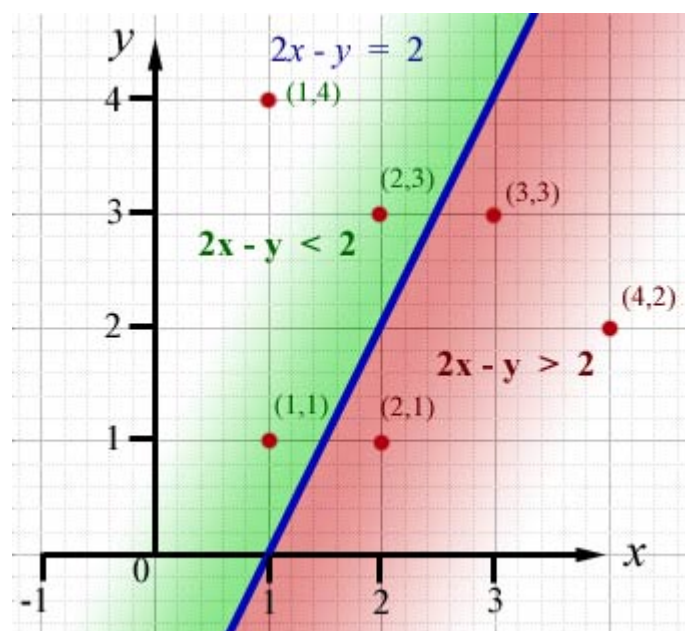
$$Ax + By = C$$

Then, above the line of the equation, $Ax + By < C$

and below the line, $Ax + By > C$

Consider the graph of $-2x + y = -2$

**note** - the first term **A** must be made positive by multiplying the whole equation by -1

The equation **-2x + y = -2** becomes **2x - y =2**



look at the points(red) and the value of **2x - y** for each. The table below summarises the result.

| point(x,y) | 2x - y | value | more than 2 ? | above/below |
|:---:|:---:|:---:|:---:|:---:|
|  |  |  |  |  |
| (1,1) | 2(1)1(1) | 1 | no - less | **above** |
| (1,4) | 2(1)-(4) | -2 | no - less | above |
| (2,3) | 2(2)-(3) | 1 | no - less | above |
| (3,3) | 2(3)-(3) | 3 | yes - more | **below** |
| (2,1) | 2(2)-(1) | 3 | yes - more | **below** |
| (4,2) | 2(4)-(2) | 6 | yes - more | **below** |

<u>The Modulus</u>

The **modulus** is the <u>numerical value</u> of a number, irrespective of the sign it carries.

hence    **| $X$ | < 3**    means    **-3 < $X$ < 3**

<u>Example</u>

$$|2x+1| < 5$$

$$\Rightarrow \quad -5 < 2x+1 < 5$$

$$-5 < 2x+1$$
$$-5-1 < 2x$$
$$-6 < 2x$$
$$\underline{-3 < x}$$

$$2x+1 < 5$$
$$2x < 5-1$$
$$2x < 4$$
$$\underline{x < 2}$$

$$\underline{-3 < x < 2}$$