<u>Algebra : Indices</u>

<u>The Laws of Indices</u>

$$p^m \times p^n = p^{m+n}$$

$$\frac{p^m}{p^n} = p^{m-n}$$

$$(p^m)^n = p^{m \times n} = p^{mn}$$

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

$$\frac{1}{p^m} = p^{-m} \qquad\qquad p^n = \frac{1}{p^{-n}}$$

$$p^0 = 1$$

<u>Indices - Multiplication</u>

remembering that:

$$p^m \times p^n = p^{m+n}$$

<u>Examples</u>

$$a^2 \times a^5 = a^7$$
$$a^{-2}b^3 \times a^5 b^{-4} = a^3 b^{-1}$$
$$a^3 b^2 \times a^4 b^7 = a^7 b^9$$
$$2a^3 b^2 c^{-3} \times 5a^3 b^{-2} c^2 = 10a^6 b^0 c^{-1} = 10a^6 c^{-1}$$
$$(b^0 = 1)$$
$$5a^2 b^{-7} c^{-2} \times 6a^{-2} b^5 c^3 = 30a^0 b^{-2} c^1 = 30b^{-2} c$$
$$(a^0 = 1, \ c^1 = c)$$

Indices - Division

remembering that:

$$\frac{p^m}{p^n} = p^{m-n}$$

Examples:

$$\frac{a^5}{a^6} = a^{-1}$$

$$\frac{a^6 b^3}{a^7 b^{-5}} = a^{-1} b^{3-(-5)} = a^{-1} b^8$$

$$\frac{a^3 b^5}{a^2 b^7} = a^1 b^{-2} = ab^{-2}$$

$$\frac{12 a^3 b^2}{3 a^{-4} b^4} = 4 a^{3-(-4)} b^{-2} = 4 a^7 b^{-2}$$

$$\frac{8 a^4 b^2 c^{-7}}{2 a^3 b^{-4} c^{-5}} = 4 a^1 b^{2-(-4)} c^{-7-(-5)} = 4 ab^6 c^{-2}$$

<u>Indices - Powers</u>

remembering that:

$$(p^m)^n = p^{m \times n} = p^{mn}$$

<u>Examples:</u>

$$(a^3 b^5)^3 = a^9 b^{15} \qquad (a^4 b^2)^{-5} = a^{-20} b^{-10}$$

$$(a^{-2} b^4)^{-3} = a^6 b^{-12} \qquad (4ab^3)^2 = 16 a^2 b^6$$

$$(3a^2 b)^3 = 27 a^6 b^3 \qquad (2a^3 b^4)^3 = 8 a^9 b^{12}$$

$$4(2ab^2)^3 = 4(8 a^3 b^6) = 32 a^3 b^6$$

$$3(4a^4 b^3)^2 = 3(16 a^8 b^6) = 48 a^8 b^6$$

<u>Indices - Roots and Reciprocals</u>

remembering that:

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

and

$$\frac{1}{p^m} = p^{-m} \qquad\qquad p^n = \frac{1}{p^{-n}}$$

<u>Examples:</u>

$$\frac{a^3}{b^4} = a^3 b^{-4}$$

$$\frac{a^2}{b^{-3} c^2} = a^2 b^3 c^{-2}$$

$$\frac{ab^2 c^3}{b^{-5} c^2} = ab^7 c$$

$$\sqrt[3]{\frac{b^4}{a^{-7}}} = \left(a^7 b^4 \right)^{\frac{1}{3}} = a^{\frac{7}{3}} b^{\frac{4}{3}}$$

$$\sqrt{\frac{a^2}{b^6}} = \left(a^2 b^{-6} \right)^{\frac{1}{2}} = a^{\frac{2}{2}} b^{-\frac{6}{2}} = ab^{-3}$$

$$\frac{a^2 b^{\frac{1}{3}}}{a^{\frac{1}{2}} b^3} = a^2 \, a^{-\frac{1}{2}} \, b^{-3} \, b^{\frac{1}{3}}$$

$$= a^{\frac{4}{2}} \, a^{-\frac{1}{2}} \, b^{-\frac{9}{3}} \, b^{\frac{1}{3}} = a^{\frac{3}{2}} \, b^{-\frac{8}{3}}$$